Case study: Acoustic attenuation performance of circular expansion chambers with extended end-inlet and side-outlet

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The acoustic attenuation performance of circular expansion chambers with extended end-inlet and side-outlet is investigated. Three approaches are employed to determine the transmission loss: (1) a one-dimensional analytical approach based on axial plane wave propagation for long chambers and diametral plane wave propagation for short chambers; (2) a three-dimensional computational technique based on the boundary element method (BEM); and (3) experiments on an extended impedance tube set-up. The transmission loss results from the BEM show good agreement with the experiments for the selected configurations over the frequency range of interest, whereas the plane wave theory may be used for a reasonable estimate at low frequencies. The effect of chamber length and relative inlet/outlet location on multidimensional wave propagation and acoustic attenuation performance of expansion chambers with end-inlet and side-outlet is discussed in detail. © 2000 Institute of Noise Control Engineering.

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1. INTRODUCTION

A number of analytical, computational, and experimental studies have investigated the effect of geometry on the acoustic attenuation performance of simple expansion chambers,\(^1\)\(^4\)\(^5\) chambers with extended inlet/outlet,\(^6\)\(^7\) and flow-reversing chambers,\(^8\)\(^9\)\(^10\) all with an end-inlet and an end-outlet. The expansion chamber with end-inlet/side-outlet (or side-inlet/end-outlet) is another important configuration used in the exhaust system of reciprocating engines. Yi and Lee developed a three-dimensional analytical approach for circular expansion chambers with side-inlet/side-outlet\(^11\) and side-inlet/end-outlet.\(^12\) Their analytical results matched the experimental transmission loss reasonably well over a wide frequency range. The chamber is modeled as a piston-driven circular rigid cylinder and the sound pressure on the inlet and outlet are averaged over the cross-sectional area. Thus, their choice facilitates a proper analysis of these configurations in the absence of connected ducts. Later, Munjal developed the four-pole parameters of these chambers based on 1-D axial wave propagation and compared the transmission loss results with those of Yi and Lee’s 3-D model for two long chambers.\(^13\) Munjal also provided the useful observation that long chambers with side inlet/outlet behave similar to extended inlet/outlet configurations, while excluding the short chambers from the study. More recently, expansion chambers with extended end-inlet and side-outlet were described by Lai, who compared the transfer function results from the modal expansion method and experiment for two specific configurations with inlet-extensions.\(^14\)

For end-inlet/side-outlet configurations, the effect of the relative location of these ducts on the transmission loss remains to be investigated in detail. The present study considers both long and short chambers with end-inlet and side-outlet ducts, including the extension of the inlet duct into the chamber. The objective is then (1) to investigate computationally by using the boundary element method, and experimentally the effect of geometry on the acoustic attenuation, with particular emphasis on the relative orientation of the inlet and outlet ducts; and (2) to present simple, one-dimensional analytical approaches to estimate the transmission loss and discuss their applicability to both types of chambers. Viscous effects are neglected in both analytical and computational studies, while mean flow is excluded from all approaches.

Following the introduction, Section 2 presents two one-dimensional analytical approaches based on axial propagation for long chambers and diametral propagation for short chambers (no extension). Section 3 compares transmission loss results from the 1-D analytical, 3-D computational and experimental approaches, and discusses the effect of geometry on multidimensional wave propagation and acoustic attenuation performance of expansion chambers with end-inlet and side-outlet. Section 4 presents concluding remarks.

2. ONE-DIMENSIONAL ANALYTICAL APPROACHES

The generation and propagation of multidimensional waves in expansion chambers is dependent on the wavelength and the dimensions of chambers (diameter, length to diameter ratio, inlet/outlet locations). At low frequencies, the effect of multidimensional waves on acoustic attenuation performance of expansion chambers is, in general, expected to be limited, and simple 1-D plane wave propagation theory may be used to obtain reasonable estimates of the attenuation behavior. Below the cut-off frequency of the first higher mode, in circular expansion chambers with end-inlet/end-outlet, a nearly one-dimensional wave propagation prevails along the diametral direction for configurations with a length to diameter ratio \(l/d\) \(<0.41\) for the concentric inlet case and \(l/d<0.853\) for the offset inlet case, respectively.\(^15\) Above these values, axial wave propagation begins to dominate. These particular ratios, \(l/d = 0.41\) for the concentric inlet case and \(l/d = 0.853\) for the offset inlet case, may be used here to identify the “short” and “long” chambers with end-inlet/side-outlet.
A. Axial propagation model

For a long expansion chamber with extended end-inlet and side-outlet (as shown in Fig. 1), axial plane wave propagation is expected to dominate at low frequencies. In order to derive the four-pole parameters, the chamber is divided at the interfaces I and II into three sections. Assuming plane wave propagation in the axial direction and using the continuity conditions of the acoustic pressure and volume velocity at inlet 1 and outlet 2 leads to

\[
\begin{bmatrix}
P_1 \\
\rho c U_1
\end{bmatrix} = \begin{bmatrix}
AB & P_2 \\
CD & \rho c U_2
\end{bmatrix}
\]  

where

\[
\begin{bmatrix}
AB \\
CD
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
\frac{j(S_o / S_1) \tan kl_0}{(S_v / S_1)} & 1
\end{bmatrix} X \begin{bmatrix}
\cos kl_b & j \sin kl_b \\
j \sin kl_b & \cos kl_b
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
\frac{j(S_v / S_2) \tan kl_v}{(S_o / S_2)}
\end{bmatrix}
\]

\[
TL = 20 \log_{10} \left( \frac{1}{2} |A + B + C + D| \right) + 10 \log_{10} \left( \frac{S_v}{S_o} \right)
\]

B. Diametral propagation model

At low frequencies, short chambers (no extension) reveal a nearly one-dimensional wave propagation along the diametral direction, as opposed to long chambers showing planar wave propagation in the axial direction. By using a segmentation approach, the four-pole parameters of short chambers with end-inlet/side-outlet may be obtained and then used to determine the transmission loss. Similar to the long chamber, the short chamber of Fig. 2 is divided at interface I into two sections designated by \( a \) and \( b \). Assuming plane wave propagation in the diametral direction and using the continuity conditions of the acoustic pressure and volume velocity at inlet 1 and outlet 2 lead to

\[
\text{Fig. 2 – Short expansion chamber with end-inlet and side-outlet.}
\]
Fig. 3 – Transmission loss of long chamber (l = 282.3 mm, offset end-inlet with $\delta_0 = 31.0$ mm and $\theta_i = 180^\circ$, $l_i = 0$, side-outlet). (a) $l_i = 80.0$ mm; (b) $l_i = 141.2$ mm; (c) $l_i = 202.3$ mm; solid line, experimental; dashed line, BEM; dotted line, 1-D analytical.

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
(S_i/S_a)/Z_a & S_i/S_a
\end{bmatrix}
\]

\[
\begin{bmatrix}
T^a_{11} & T^a_{12} \\
T^b_{21} & T^b_{22}
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & S_2/S_\Pi
\end{bmatrix}
\]

(4)

where $Z_a = -P_a/(\rho c U_j)$ is the specific acoustic impedance of the section $a$; $S_1$ and $S_\Pi$ are the cross-sectional areas of the plane interfaces I and $\Pi$, and $T^a$ and $T^b$ are the four-pole parameters of sections $a$ and $b$, respectively. To determine $T^a_{ij}$ and $T^b_{ij}$, the sections $a$ and $b$ are modeled as a series of conical duct segments with a rectangular cross-section. The four-pole parameters of an arbitrary conical segment $(i)$ may then be obtained following the method of ref. 18. The transfer matrix of a structure consisting of, for example, $n$ conical segments are determined by $[T] = \Pi^a [T]^b$ which are subsequently used in Eq. (4). Once the four-pole parameters are determined, the transmission loss can then be calculated by using Eq. (3).
3. RESULTS AND DISCUSSION

For all configurations, the present study considers \( d = 153.2 \) mm for the chamber diameter, \( d_i = d_o = 48.6 \) mm for the inlet/outlet ducts. The speed of sound in experiments and computations is 346 m/s. Long chambers with no inlet-extension (\( l_i = 0 \) in Fig. 1) are considered first. To examine the effect of relative inlet/outlet locations on acoustic attenuation performance, Figs. 3-6 present the transmission loss of a long chamber (\( l = 282.3 \) mm) for four different inlet locations – Fig. 3: offset with \( \delta_i = 51.0 \) mm and \( \Theta_i = 180^\circ \); Fig. 4: offset with \( \delta_i = 51.0 \) mm and \( \Theta_i = 0^\circ \); Fig. 5: offset with \( \delta_i = 51.0 \) mm and \( \Theta_i = 90^\circ \); and Fig. 6: concentric. Each figure consists of three graphs corresponding to three different outlet locations: (a) \( l_o = 80 \) mm (closer to the second end plate); (b) \( l_o = 141.2 \) mm (middle); and (c) \( l_o = 202.3 \) mm (closer to the first end plate or inlet plate). Each figure compares the transmission loss from the boundary element predictions with the experimental results. The experimental setup for the extended impedance tube has been discussed elsewhere. Also included are the predictions from the simple 1-D approach to assess its applicability. The BEM calculations are extended 0.10 m into both inlet and outlet ducts. The boundary surfaces are discretized into eight-node quadrilateral and six-node triangular elements with the longest size being smaller than 1/6 wavelength at highest frequency. Further details of the
Fig. 7 – Transmission loss of long chamber \( l = 282.3 \text{ mm}, \) concentric end-inlet, \( l_e = 0 \); solid line, side-outlet with \( l_s = 141.2 \text{ mm} \); dashed line, concentric end-outlet with no extension.

Numerical procedure using quadratic isoparametric elements for three-dimensional analysis can be found elsewhere. The boundary element predictions of transmission loss in Figs. 3-6 show good agreement with experiments, while the axial 1-D solution reveals reasonable estimates at low frequencies, particularly below the first higher order mode \((1, 0)\) of chamber. Figure 3 with \( 180^\circ \) offset inlet exhibits a behavior similar to Fig. 4 with \( 0^\circ \) offset inlet, except that the attenuation dome between 600 Hz and 1300 Hz in Fig. 3c is replaced by a broad resonance peak in Fig. 4c, which improves the acoustic performance. The transmission loss of long chambers with end-inlet/side-outlet exhibit a superposition of domes and resonance peaks in the plane wave region. The resonances are due to the end cavity between the outlet and the right endplate. With the outlet approaching the inlet by increasing the length \( l_s \), the number of resonance peaks increases and the resonance frequencies are lowered (for example, note the first resonance, \( f_1 = 1010 \text{ Hz in Fig. 3a}, f_1 = 610 \text{ Hz in Fig. 3b}, f_1 = 450 \text{ Hz in Fig. 3c} \). The outlet location may be chosen such that the resonances are located at zero-attenuation frequencies of expansion chambers leading to a desirable high acoustic attenuation. For example, by locating the outlet at the center of the expansion chamber, the odd troughs of expansion chambers can be eliminated as illustrated in Figs. 3b, 4b, 5b, and 6b, which demonstrates the benefit of the side-outlet in silencer design.

Similar to the simple expansion chamber with end-inlet/ end-outlet, the offset inlet will excite asymmetric higher order modes and can significantly affect the acoustic attenuation of expansion chambers with end-inlet/side-outlet. For the \( 180^\circ \) or \( 0^\circ \) offset inlet, an effective attenuation region is observed until the first asymmetric \((1, 0)\) mode, as shown in Figs. 3a-c and 4a-c. Beyond this mode, the transmission loss of the expansion chamber is reduced markedly. The detrimental effect

Fig. 8 – Transmission loss of long chamber \( l = 282.3 \text{ mm}, \) offset end-inlet with \( \delta_1 = 51.0 \text{ mm and } \theta_1 = 180^\circ, l_e = 0 \); solid line, side-outlet with \( l_s = 80.0 \text{ mm} \); dashed line, extended end-outlet with \( l_e = 80.0 \text{ mm} \).

Fig. 9 – Transmission loss of long chamber \( l = 282.3 \text{ mm}, \) concentric end-inlet, side-outlet with \( l_s = 141.2 \text{ mm} \); (a) \( l_e = 80.0 \text{ mm} \); (b) \( l_e = 141.1 \text{ mm} \); (c) \( l_e = 202.3 \text{ mm} \); solid line, BEM; dotted line, 1-D analytical.
of the asymmetric (1,0) mode could be partially eliminated by placing the inlet and outlet 90° apart. The results shown in Fig. 5a-c exhibit increased transmission loss over the 180° and 0° cases specifically in the frequency range between the (1,0) and (2,0) modes, combined with some effect beyond the (2,0) mode. Also similar to expansion chambers with end-inlet/end-outlet, centering the inlet can improve the acoustic attenuation approximately till the first higher order radial (0,1) mode of the chamber, as illustrated in Fig. 6 in comparison with Figs. 3, 4, and 5.

It is interesting to compare the side-outlet configuration of, for example, Fig. 6b to the concentric end-outlet chamber of Fig. 14 in reference 4, while retaining the same dimensions. The transmission loss comparison for these two configurations based on the BEM is shown in Fig. 7. The right end cavity introduces an additional resonance peak at the pass-band in between two attenuation domes of the chamber with concentric end-outlet. The superposition of two domes and the resonance peak forms a broader-dome-like behavior. The two domes of the end-outlet are now replaced by one major dome of the side-outlet, which matches essentially an acoustically halved length, but with higher attenuation.

The behavior of side-outlet versus extended end-outlet configurations (same chamber length and diameter as well as the inlet location, and the length $l_f$ for the side outlet being the same as that of the extended outlet duct) is compared in terms of the 3-D BEM computational results in Fig. 8. The qualitative agreement is substantial, which is consistent with Munjal's observations on a chamber with significantly different acoustic characteristics.16

Figure 9 illustrates the effect of an extended inlet duct for a configuration with the outlet fixed in the middle of the chamber. In comparison with the chambers without inlet extension (Fig. 5b), this configuration reveals a similar superposition of attenuation domes and resonance peaks in the plane wave region, except the inlet extension now introduces additional resonance peaks. As the extended inlet approaches the outlet, the resonance frequency due to the inlet extension is lowered for the fixed outlet case (Note, for example, $f_i = 890$ Hz for Fig. 9a, and $f_i = 770$ Hz for Fig. 9b). Further extending the inlet into the chamber coupled with the side outlet creates a rather desirable wide-band attenuation, as depicted in Fig. 9c. While the BEM predictions reveal that the long extension (Fig. 9c) provides a better attenuation than the short one (Fig. 9a), the 1-D predictions do not exactly capture this trend. The 1-D approach does, however, reasonably well at lower frequencies.

For short expansion chambers with end-inlet/side-outlet, a configuration with $l = 48.6$ mm is considered here. Figures 10a for the 180° offset end-inlet, 10b for the 0° offset end-inlet, 10c for the 90° offset end-inlet, and 10d for the concentric end-inlet compare the transmission loss results from the 1-D diametral model and the BEM, which exhibit a trendwise agreement at low frequencies. With increasing frequency, multidimensional waves begin to dominate for the short chamber, terminating the applicability of the 1-D diametral model. The number of segments for the transmission loss calculation using the 1-D diametral model for such short chambers has been discussed elsewhere.18 Also included in Fig. 10 are the results from the 1-D axial predictions for

Fig. 10 — Transmission loss of short chamber ($l = 48.6$ mm, end-inlet with no extension, side-outlet with $l_f = 24.3$ mm). (a) offset end-inlet with $\delta_i = 51.0$ mm and $\theta_i = 180^\circ$, (b) offset end-inlet with $\delta_i = 51.0$ mm and $\theta_i = 0^\circ$, (c) offset end-inlet with $\delta_i = 51.0$ mm and $\theta_i = 90^\circ$, (d) concentric end-inlet: solid line, BEM; dashed line, 1-D diametral; dotted line, 1-D axial.
comparison purposes. The short expansion chambers with end-
inlet/side-outlet of Fig. 10 clearly show no similarity between
the 1-D axial and 3-D predictions due to dominant transverse
propagation. Figure 10a for the 180° offset inlet resembles the
dome-like (wide band) behavior due to the one-dimensional
propagation in the transverse direction of the chamber with
a smaller effective expansion ratio, whereas Figs. 10b for the
0° offset inlet, 10c for the 90° offset inlet, and 10d for the
concentric inlet exhibit predominantly peak behavior, typical
of side-branch resonators.

4. CONCLUDING REMARKS

The acoustic attenuation performance of circular expansion
chambers with extended end-inlet and side-outlet is
investigated analytically and computationally for both long and
short chambers, as well as experimentally for long chambers
with no extension. Accurate prediction at higher frequencies
requires a multidimensional approach, while the simple one-
dimensional theory (1-D axial model for long chambers and
1-D diametral model for short chambers) may be employed
for reasonable estimates at low frequencies. The long chambers
with end-inlet (no extension) and side-outlet exhibit the
combination of broadband attenuation domes and resonance
peaks below the cut-off frequency of the first excited higher
order mode of the chamber. The number of resonance peaks
increases and the resonance frequencies decrease as the length
l, is increased. An effective attenuation band is observed until
the first asymmetric (1,0) mode for the 180° and 0° offset inlet.
Placing the inlet and outlet 90° apart can improve the
transmission loss over the 180° and 0° cases, in the frequency
range between the (1,0) and (2,0) modes. Centering the inlet
extends the attenuation until the first radial (0,1) mode, which
demonstrates the benefit of the concentric inlet arrangement
in the design. Beyond the cut-off frequencies of these modes
[(1,0) mode for the 180° and 0° offset inlet, (2,0) mode for the
90° offset inlet, and (0,1) mode for the concentric inlet], the
attenuation, in general, is reduced. Long chambers with inlet-
extension exhibit similar superposition of domes and resonance
peaks to the chambers without inlet extension, except that the
extension introduces additional resonance peaks. By choosing
the locations of the inlet and outlet to match the resonances
with the pass-band frequencies of the expansion chambers,
excellent acoustic attenuation may be obtained. The short
chambers exhibit dominant transverse propagation, with the
characteristics being significantly affected by the orientation
of the end-inlet.

5. REFERENCES

1. A.E. El-Sharkawy and A.H. Nayfeh, “Effect of the expansion chamber on
the propagation of sound in circular pipes,” J. Acoust. Soc. Am. 63, 667-
672 (1978).
2. L.J. Eriksson, “Higher order mode effects in the circular ducts and expansion
3. L.J. Eriksson, “Effect of inlet/outlet locations on higher order modes in
4. A. Selamat and P.M. Radavich, “The effect of length on the acoustic
attenuation performance of concentric expansion chambers: an analytical,
computational, and experimental investigation,” J. Sound Vib. 201, 407-
426 (1997).
8. J. Kim and W. Soedel, “General formulation of four pole parameters for
three-dimensional cavities utilizing modal expansion, with special attention
212 (1999).
11. A. Selamat and Z.L. Ji, “Acoustic attenuation performance of circular flow-
13. C.-L. J. Young and M.J. Crocker, “Acoustical analysis, testing and design of
18. A. Selamat and Z.L. Ji, “Diametral plane wave analysis for short circular
to acoustic cavity response and muffler analysis,” J. Vib. Acoust. Stress
21. Z.L. Ji, Q. Ma, and Z.H. Zhang, “Application of the boundary element method
to predicting acoustic performance of expansion chamber mufflers