AAE 5620: Stability and Control of Flight Vehicles

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Introduction

- The objective of this course is to perform an intermediate analysis of dynamics, stability and control of atmospheric flight vehicles. The course title is somewhat ambiguous in the sense it doesn't clarify the class of flight vehicles it is concerned with. In keeping with the tradition of this course, we will only consider aircraft.
- Notwithstanding the favoritism to aircraft, the first quarter of the course contain material on dynamics and small perturbation theory that is applicable to both air- and spacecraft. In particular:

  Topics in **dynamics:**

  - Reference frames; rotation matrices. Motion of an object idealized as a particle; transport theorem.
  - Representation of attitude. Motion of an object idealized as a rigid body; Eulerian framework.

  Topics in **small perturbation theory:**

  - Linearization of equations of motion; assumptions and domain of validity.
  - Notion of static stability and meaning of stability derivatives.

  Then we will go through topics that are more specific to aircraft, using throughout linearized equations of aircraft motion:

  - Identificaiton of **flight modes:** essentially an exercise is *order reduction* via decoupling.
  - **Dynamic stability** of flight modes.
  - **Stability and control augmentation** systems using fundamental concepts of feedback control theory.
  - **Autopilot design** for response holds and attitude control.
- Our first lesson on stability and control of aircraft comes from a lecture delivered by Wilbur Wright, to the Western Society of Engineers in the year 1901. Below is an excerpt:

  "The difficulties which obstruct the pathway to success in flying-machine construction are of three general classes:

  1. *Those which relate to the construction of the sustaining wings;*
  2. *Those which relate to the generation and application of the power required to drive the machine through the air;*
  3. ***Those relating to the balancing and steering of the machine after it is actually in flight***

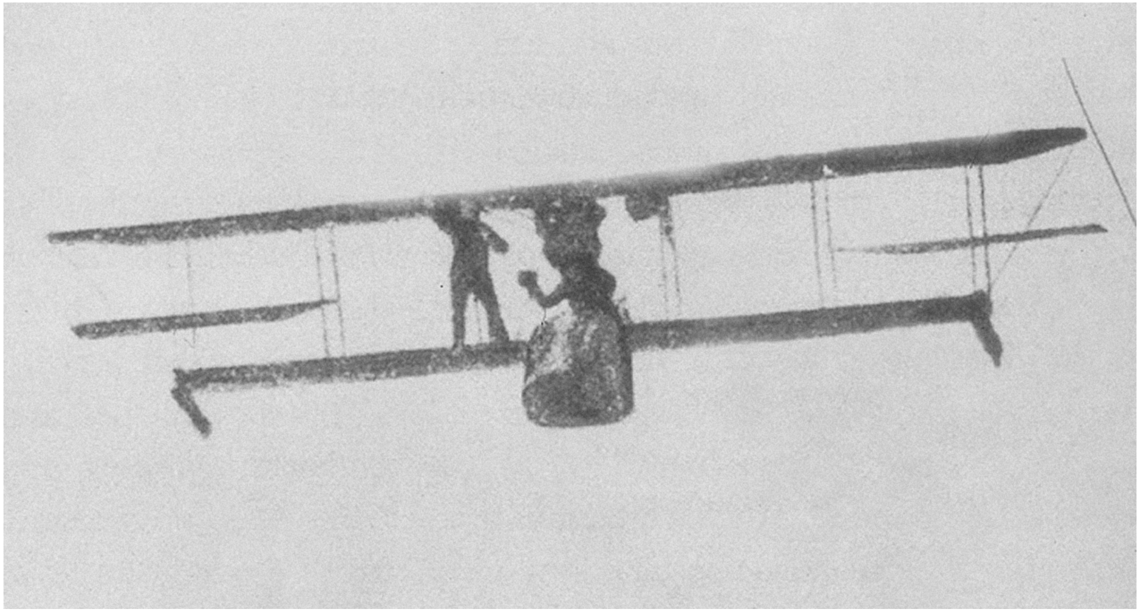


Figure 1: Lawrence Sperry's Demonstration of the Roll + Pitch Attitude Stabilizer. Paris, 1914: Aero Club of Frame, Aeroplane Safety Competition.

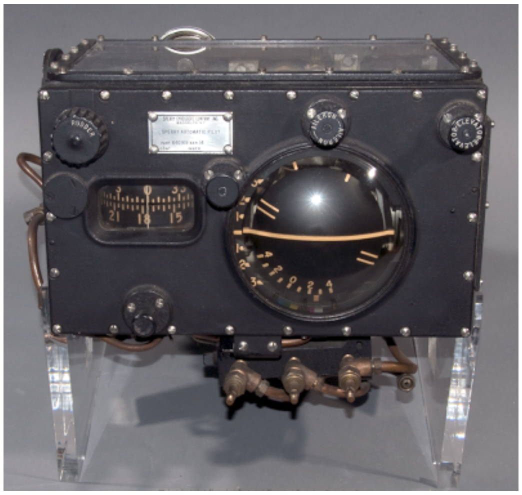
> *Of these difficulties two are already to a certain extent solved. Men already know how to construct wings or aeroplanes which, when driven through the air at sufficient speed, will not only sustain the weight of the wings themselves, but also that of the engine and of the engineer as well. Men also know how to build engines and screws of suffcient lightness and power to drive these planes at sustaining speed. As long ago as 1884 a machine weighing 8,000 pounds demonstrated its power both to lift itself from the ground and to maintain a speed of from 30 to 40 miles per hour,* ***but failed of success owing to the inability to balance and steer it properly. This inability to balance and steer still confronts students of the flying problem, although nearly eight years have passed. When this one feature has been worked out, the age of flying machines will have arrived, for all other difficulties are of minor importance."***

- Research activity in automatic flight control of fixed wing aircraft can actually be traced as far back as 1873, when Col. Charles Renard used a pair of steering wings controlled to a transverse pendulum that could make them rotate in a differential manner. The technology was used on Renard's "decaplane" (ten wings!), which unfortunately did not fly.

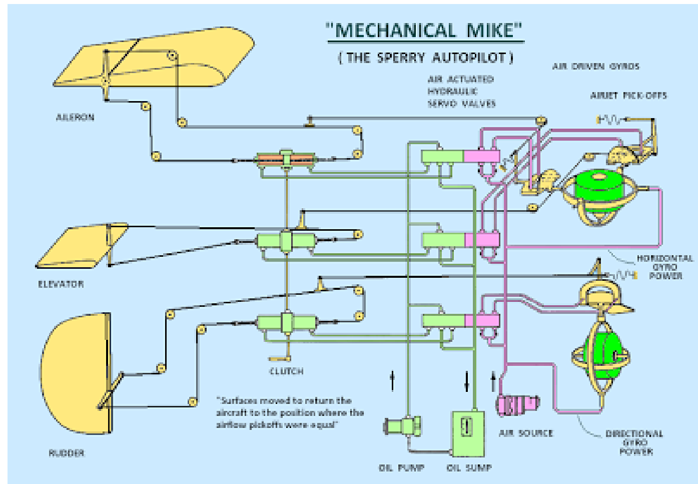
- Elmer Sperry is credited with the invention of the fist autopilot system in 1912. It was a gyroscopic stabilizer with only lateral control. Later, in 1914, a dramatic exhibition was given by Sperry Gyroscope Co. at the "Aeroplane Safety Competition" organized by the Aero Club of France, in which Lawrence Sperry (Elmer's son) stood in the cockpit with his hands above his head and his French mechanic, Emile Cachin walked on the aircraft wing with the aircraft in steady low level flight: Fig.(1). The 1914 autopilot provided roll and pitch stabilization. In each direction, the attitude was sensed by counterrotating gyros. Attitude errors operated mechanical roller switches which in turn actuated pneumatic servos to move the elevators and ailerons.

- The Sperry autopilot design continued to improve. Figs.(2) shows the $A3$ autopilot system, also known as Mechanical Mike. The predecessor of $A3$, the $A2$ was used on Wiley Post's Lockheed Vega, named "Winnie Mae", on which he flew solo around the world in 7 days, 18 hours and 49 minutes in 1933.

- The first *all-automatic* flight from take-off to landing was achieved in September 1947, by a four-engine USAF $C-54$ Skymaster. It had a Sperry $A12$ autopilot. The New York Times hailed it as a "triumph of automatic control."



(a) The Sperry $A3$ Autopilot: Mechanical Mike, *circa* 1933.



(b) A Schematic of Mechanical Mike

Figure 2: The State of the Art in Autopilot Systems in the Early 1930's: Sperry $A3$: aka Mechanical Mike

- For more details on the early development of flight stabilization and control, see Howard's article (Ref 1.) on the fist 100 years of flight controls.
- There are two key concepts considered in this course **Stability** and **Control**. Below, we browse through initial thoughts on each one of them.

Stability

- *Stability* is a crucial concept around which several key ideas of feedback control are built.
- Often, we talk about the "stability of systems"; but this is fundamentally incorrect. Stability is a property of *reference states*: not of systems. Indeed, the exact same system can be stable and unstable about different reference points: see Fig.(3). A simple pendulum, when hung in its "normal configuration", shown to the left, is well known to be stable. Let us call this configuration $\mathcal{C}_0$, which represents the mass located at a displacement of 0 deg from the vertical. Our common sense understanding of stability tells us that the mass *tends to return to its original configuration, i.e.* $\mathcal{C}_0$*, if a "reasonable disturbance" displaces it, as shown.*

  On the other hand, the same pendulum, when in configuration $\mathcal{C}_\pi$, which represents the mass displaced at an angle of π from the vertical is known to be unstable. In the absence of any disturbances, the pendulum will maintain this precarious configuration forever. However, the slightest disturbance causes it to exhibit ever increasing deviation from its reference position $\mathcal{C}_p i$.

  The moral of the story is that it is not the pendulum that is stable or unstable. The property of stability must instead be bestowed upon its two reference states, $\mathcal{C}_0$ and $\mathcal{C}_\pi$, which are stable, and unstable, respectively.
- In the example above, the "reference configuration" was taken to be static, i.e. not dynamically changing (often called a *fixed point*). But this is not necessary. The idea of stability can be easily extended to "reference trajectories", such that a reference trajectory is stable if disturbances about it are suppressed and unstable if not.

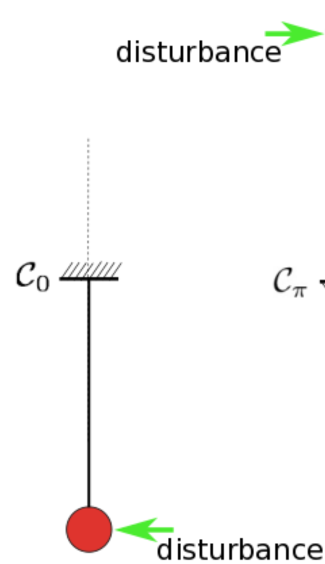


Figure 3: Is the Pendulum Stable or Unstable? Depends on its Reference Orientation!

- The early development of aircraft was driven by the ultimate need to make its structure have inherently stable reference points. This was deemed important in order to relieve the pilot from constantly having to flight disturbances hitting the aircraft, such as wind gusts. This lead to an fascinating struggle - because as you can imagine, a highly stable reference must also be highly *stiff*. In other words, the "greater the stability" of a reference, the greater the effort required must be to move the system away from it, because of the inherent tendency of the system to return to the said reference.

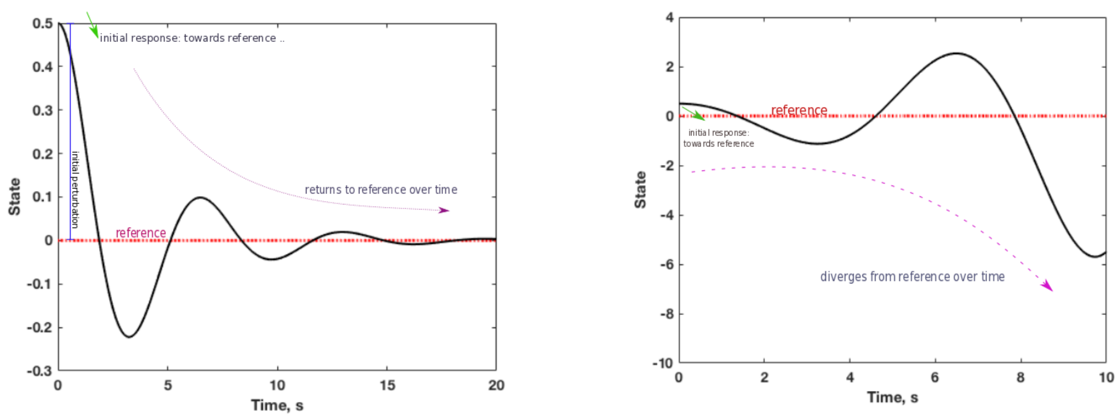
  Clearly, while stability is desirable, *too much stability* can become a nuisance because the aircraft does not operate around a single reference point throughout its flight. Moreover, stability is directly at odds with maneuverability, which is an absolute necessity for fighter aircraft. Indeed, many fighted jets are designed to be inherently unstable around several reference points. This means two things: (i.) they are easy to maneuver, but, (ii.) they cannot fly without the assistance of automatic feedback control systems that keep it from moving dangerously away from safe flying configurations.

- There are two fundamental notions of stability that we are interested in:

  1. **Static Stability.** This notion captures the initial tendency or the first response of a system in the event of a disturbance from its the reference state. If the system shows an "inclination" to return to its reference, the reference is said to be statically stable, e.g. the pendulum on the left in Fig.(3). On the other hand, if the initial inclination is to deviate further from the reference, it is said to be statically unstable. Such a reference is shown on the right in Fig.(3). As one can expect, static stability is the minimum requirement (a necessary condition) for any type of stability.
  2. **Dynamic Stability.** This notion captures what happens in due course after the perturbation hits the system. Clearly, dynamic stability analysis is much more detailed and studies the time-varying characteristics of the system as it eventually either returns to its reference state, or doesn't. In the former case, the reference is said to be dynamically stable and in the latter, dynamically unstable. A commonly used notion of dynamic stability in the field of feedback control is **BIBO**

stability. BIBO stands for "Bounded Input Bounded Output". A reference is BIBO stable if for every bounded input signal (think disturbance) the output signal is also bounded.

- Fig.(4) illustrates the notions of static and dynamic stability. The two figures represent <u>different systems.</u> In both cases however, the reference under consideration is the origin, call it $x = 0$. An initial perturbation is shown of 0.5 units, such that the system is thrown off the reference position at t_0. The initial tendency of both systems is to return to the origin, as is clearly seen in the two figures. In other words, each system is *statically stable* about the shown reference. However, only the reference state on the left is the only one that is also *dynamically stable*, because the system eventually returns to it. Despite static stability, the origin is not dynamically stable for the system on the right, as is clearly illustrated by the diverging trajectory.



(a) Statically <u>and</u> Dynamically Stable　　(b) Statically Stable but Dynamically Unstable

Figure 4: The Notions of Static and Dynamic Stability of a Reference State

- Most of the popular stability analysis, both the static and dynamic versions, is conducted within the framework of *small perturbation theory*, which in turn enables the application of linear systems theory. In essence, the idea is to assume that the disturbances which cause the system to deviate from its reference states are **small**. This assumption gives us the license to first use the Taylor series expansion of the system's states about the reference of interest, and then drop all terms in the expansion except the leading linear term. The stage is set to invoke linear systems theory and the magic of modal analysis to study stability.

- To summarize, there are three ingredients in the study of stability:

  1. **A Reference.** The concept of stability is applied not to systems, but to its reference states. So, we must first identify the reference whose stability is to be analyzed. This could be a fixed point, e.g. a particular attitude to be maintained by an aircraft, or a time-varying trajectory, e.g. a pull-up maneuver to be executed at a fixed speed. As an example, consider a nonlinear dynamic system with scalar state x and reference x^* such that:

$$\dot{x} = f(x) \tag{1a}$$
$$\dot{x}^* = f(x^*) \tag{1b}$$

  2. **A Disturbance.** Stability studies the response of systems to disturbances about reference states. In order to keep things "well behaved" (i.e. ensure linearization is valid), we must assume that all perturbations and disturbances are "small". The definition of "small" is system dependent and often a matter of making educated guesses.

3. **Small Perturbation Assumption and Linear Systems Theory.** Once we identify the reference and define a disturbance, all the system states are written as a perturbation over the reference. Let us denote the disturbance as δ. So, we write the state as follows:

$$x = x^* + \delta \tag{2}$$

Eq.(2) allows the use of Taylor's expansion, in terms of the perturbation, δ, about the reference state, x^*. Substitute Eq.(2) in Eq.(1a) to get the following developments

$$\dot{x}^* + \dot{\delta} = f(x^* + \delta) \tag{3a}$$

$$\dot{x}^* + \dot{\delta} = f(x^*) + \frac{\partial f}{\partial x}\Big|_{x^*} \delta + \underbrace{\frac{1}{2}\frac{\partial^2 f}{\partial x^2}\Big|_{x^*}\delta^2 + \cdots}_{\textbf{drop... (small perturbation)}} \tag{3b}$$

$$\dot{\delta} \underset{\text{Eq.(1b)}}{=} a\delta \tag{3c}$$

where, $a = \frac{\partial f}{\partial x}\Big|_{x^*}$ is called the **stability derivative**. Note that Eq.(3c) is linear and the fate of the perturbation depends entirely on the stability derivative, a. Conclusions about both static and dynamic stability can now be derived based on linear systems theory.

Clearly, one must never forget that this was all possible because of the magic that happened in Eq.(3b), which allowed us to drop the higher order terms – the magic of "small perturbation", which justifies the inequality $|\delta|^n \ll |\delta| \;\; \forall\, n \geq 2$.

Control

The notion of feedback

- The essential idea is to have a "control system" (*the controller*) that allows a "dynamic system" (the system to be *controlled*) to follow a prescribed plan of action by computing corrective actions when the plant deviates from the plan. The corrective action is a function of the extent of deviation from the plan (*the error*). Below is a sampling of some classic texts on feedback control and how they explain the concept:

  – *Åström & Murray, Feedback Systems, Chapter 1...*

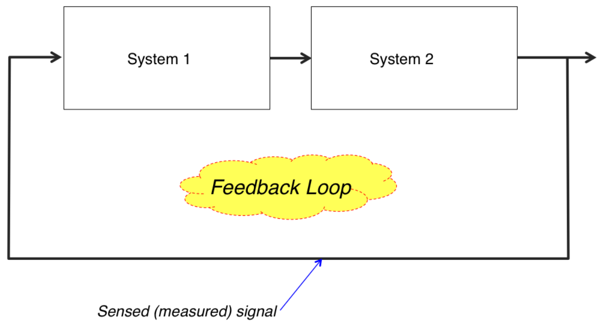
  "The term *feedback* refers to a situation in which two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled. Simple causal reasoning about a feedback system is difficult because the first system influences the second and the second system influences the first, leading to a circular argument. This makes reasoning based on cause and effect tricky, and it is necessary to analyze the system as a whole. A consequence of this is that the behavior of feedback systems is often counterintuitive, and it is therefore necessary to resort to formal methods to understand them."

  An illustration of the above; in particular, the distinction between *feedback* and *open-loop* systems is shown in Fig.(5).

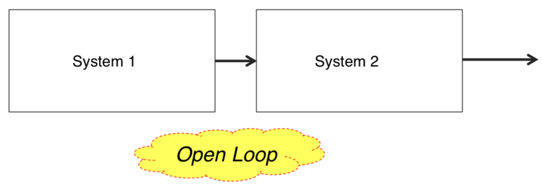
  – *Doyle, Francis & Tannenbaum, Feedback Control Theory, Chapter 1...*

  "Without control systems, there could be no manufacturing, no vehicles, no computers, no regulated environment - in short, no technology. Control systems are what make machines, in the broadest sense of the term, function as intended. Control systems are most often based on the principle of feedback, whereby the signal to be controlled is compared to a desired reference signal and the discrepancy used to compute corrective control action..."

  ..also from Chapter 3... "The most elementary feedback control system has three components: a plant (the object to be controlled, no matter what it is, is always called the *plant*, a sensor to measure the output of the plant, and a controller to generate the plant's input. Usually actuators are lumped in with the plant."



(a) Feedback system: **closed loop**



(b) **Open loop** system

Figure 5: Closed loop and open loop: an illustration of interconnections as described by Åström & Murray

– *Ogata, Feedback Systems, Chapter 1...*

"A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a *feedback control system*... Feedback control systems are not limited to engineering but can be found in various nonengineering fields as well. The human body, for instance, is a highly advanced feedback control system. Both body temperature and blood pressure are kept contact by means of physiological feedback.."

"..Feedback control systems are often referred to as *closed-loop control systems*. In practice, the terms feedback control and closed-loop control are used interchangeably In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the systems to a desired value... ..Those systems in which the output has no effect on the control action are called *open-loop control systems*. In other words, in an open-loop control system the output is neither measured nor fed back for comparison with the input. "

– *Dorf & Bishop, Modern Control Systems, Chapter 4...*

"A control system is defined as an interconnection of components forming a system that will provide a desired system response. Because this desired system response is known, a signal proportional to the error between the desired and the actual response is generated. The use of this signal to control the process results in a closed-loop sequence of operations that is called a feedback system."

- You can probably gather from above that feedback systems involve an interconnection among the following key elements:

i.) a **process** (to be controlled). Commonly also referred to as the *plant*, this is the "central object" - the entity that must be controlled, i.e. made to behave a certain way.

For us in this course, this is the **aircraft**. Of course, there is some confusion because the "aircraft" also carries the other elements of feedback listed below. In this sense, it is difficult to completely detach the controller from the plant, because the former is contained in, and moves, with the latter.

ii.) **sensors.** These are what give feedback systems their "awareness" by virtue of the measurements they make to be used as feedback. Without sensors there can be no feedback.

An aircraft carries a wide range of sensors that measure windspeed, attitude, altitude, temperature, pressure, etc. Without sensors, automatically controlled flight and all of the ideas discussed in this course are impossible. A UTC pitot probe system and a Honeywell military radar altimeter are shown in Fig.(6).

(a) A UTC SmartProbe Pitot System: Measuring air data: speed, temperature, pressure

(b) Location of Pitot Probes

(c) A Honeywell Radar Altimeter

Figure 6: Two (of the many) Sensors Used on Aircraft to Enable Automatic Feedback Control

iii.) **controller.** This is the computer, or the algorithm, to which we feed the difference between the measured signal and the externally input reference signal, i.e. the *error signal*:

$$\underbrace{e(t)}_{\text{error}} = \underbrace{r(t)}_{\text{input (reference)}} - \underbrace{y(t)}_{\text{measurement}} \tag{4}$$

The controller uses the error signal to generate control commands. Depending on how the control is generated from the error, we get different types of controllers, e.g. see PID controller in Eqs.(6) below.

A BAE Systems flight computer designed for commercial aircraft is shown below in Fig.(7)

Figure 7: A BAE Systems Flight Computer: "The Controller".

iv.) **actuators.** These are the elements that *physically implement* the control commands. In mathematical modeling, actuators are often clubbed together with the controller.

Traditional aircraft actuation is achieved via control surfaces shown in Fig.(8). Technically, the control surfaces should be called "actuator surfaces". Control surfaces are usually divided into primary and secondary systems. The ailerons, elevator (or stabilator), and rudder constitute the primary control system and are required to control an aircraft safely during flight. Wing flaps, leading edge devices, spoilers, and trim systems constitute the secondary control system and improve the performance characteristics of the airplane or relieve the pilot of excessive control forces.

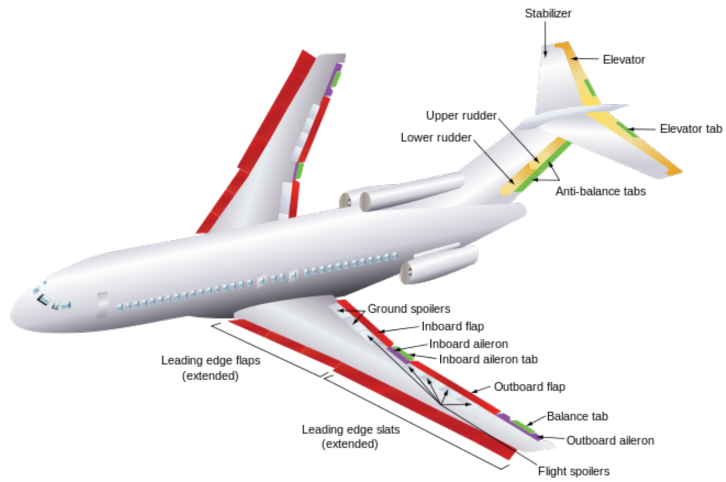


Figure 8: Actuation through Control Surfaces on a Boeing 727.

- The composition of the above systems in feedback fashion is called a *feedback control system*: see Fig.(9). Note that the measured signal is *subtracted* from the reference signal: this is called *negative feedback*. Inadvertent positive feedback often leads to poor performance, even instability.

  A diagram of the type shown in Fig.(9) is called a *block diagram.* In practice, it is common to combine the control & actuator blocks into a single block with input $e(t)$ and output $u(t)$ (control signal). This is also true in actual, physical control systems: the actions of one or several of these blocks may be performed by a single component. See examples below.

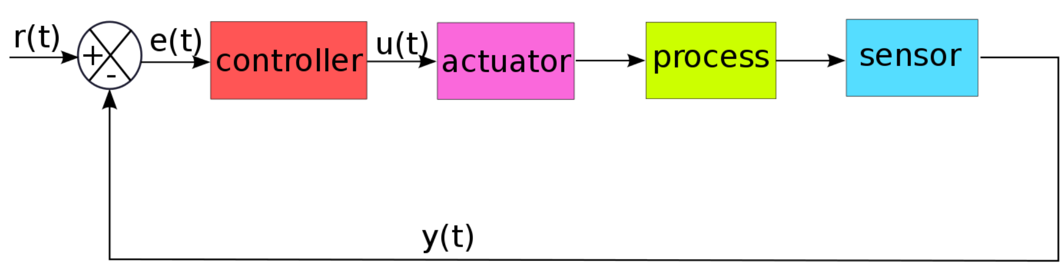
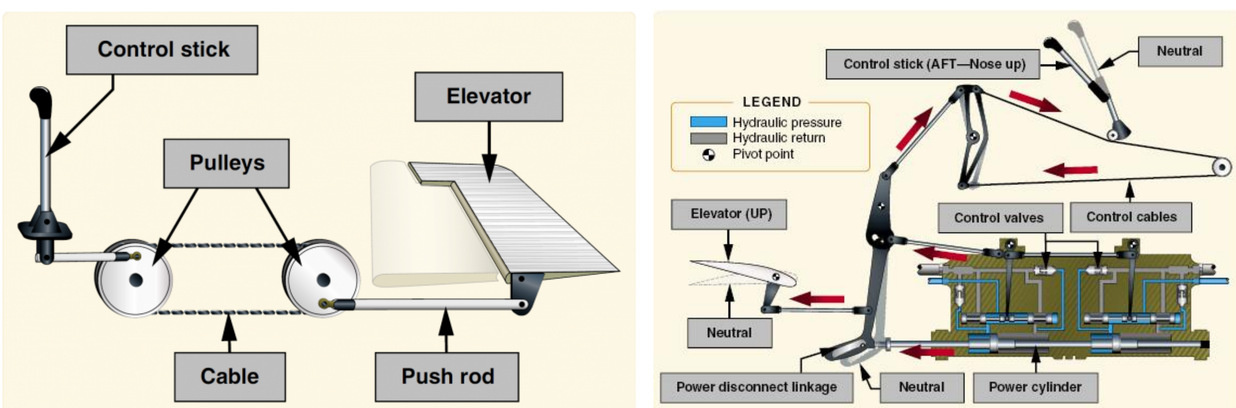


Figure 9: A feedback control system, a.k.a. closed-loop system

- There is tremendous variation in the nature of flight control elements used, depending on the type of aircraft. The most basic flight control systems are purely mechanical, and date back to early aircraft, as mentioned above. Even today, some small aircraft, typically in the sport category use purely mechanical control systems. These comprise solely of mechanical components such as rods, cables and pulleys. A schematic is shown in Fig.(10(a)). The aerodynamic forces on such aircraft are not very strong and response times are typically longer.

- On larger, faster aircraft, more complex systems are required, such as the hydromechanical flight control system shown in Fig.(10(b)). These were the first advancements over purely mechanical assemblies. With increasing sophistication, the control surfaces were actuated by electric motors, digital computers, or fiber optic cables. Called **fly-by-wire**, this flight control system (Fig.(7)) replaces the physical connection between pilot controls and the flight control surfaces with an electrical interface. In addition, in some large and fast aircraft, controls are boosted by hydraulically or electrically actuated systems. In both the fly-by-wire and boosted controls, the feel of the control reaction is given to the pilot by simulated means.
- The ***autopilot*** is an automatic flight control system that keeps an aircraft in level flight or on a set course. It can be directed by the pilot, or it may be coupled to a radio navigation signal. Autopilots reduce the physical and mental demands on a pilot and increases safety. The common features available on an autopilot are altitude and heading hold. The simplest autopilot systems use gyroscopic attitude indicators and magnetic compasses to control servos connected to the control surfaces. The number and location of these servos depends on the complexity of the system. For example, a single-axis autopilot controls the aircraft about the longitudinal axis and a servo actuates the ailerons. A three-axis autopilot controls the aircraft about the longitudinal, lateral, and vertical axes. Three different servos actuate ailerons, elevator, and rudder. More advanced systems often include a vertical speed and/or indicated airspeed hold mode. systems are coupled to navigational aids through a flight director.



(a) A Mechanical Flight Control System　　(b) A Hydro-mechanical Flight Control System

Figure 10: Mechanical and Hydromechanical FCS: Taken from FAA's Pilot's Handbook of Aviation Knowledge.

- As mentioned above, the principle of feedback is simple: provide corrective actions ($\sim$ control) based on the difference between desired and actual performance (determined by sensor). Feedback has been known to drastically improve system capability. Below we enumerate some of its key features.

  1. The main use of feedback is to provide **robustness to uncertainty**. The idea is that by measuring the difference between the sensed value of a regulated signal and its desired value, a corrective action can be supplied. This difference is often referred to as an *error signal*: which was first mentioned in Eq.(4) above. We repeat it below, with a notational variation:

$$e(t) = \underbrace{y_d(t)}_{r(t) \text{ in Eq.(4)}} - y_s(t) \tag{5}$$

In this equation, the actual value of the regulated signal is $y(t)$. Its measured (sensed) value is $y_s(t)$, which is different from the actual signal due to sensor (instrument) error that typically manifests in the form of measurement noise. The desired value is $y_d(t)$, often also written as $r(t)$, and called the *reference signal*. The widely popular PID controller is the summation of three control measures – (i) proportional (P) to the error signal, (ii) integral of the error signal, and (iii) derivative of the error signal:

$$u_{\mathtt{P}}(t) = K_P e(t) \tag{6a}$$

$$u_{\mathtt{I}}(t) = K_I \int_0^t e(\tau)d\tau \tag{6b}$$

$$u_{\mathtt{D}}(t) = K_D \frac{de}{dt}(t) \tag{6c}$$

$$u_{\mathtt{PID}}(t) = u_P(t) + u_I(t) + u_D(t) \tag{6d}$$

Systems are invariably driven by forces that are poorly understood, e.g. wind gusts hitting an airplane, bumps on a road affecting the operation of a car, etc. These forces are best modeled as "random" or "uncertain" disturbance inputs that perturb the system under study. In addition, there are sources of uncertainty within the system model, e.g. parameters whose true value may be different from their assumed value. Consider the numerous components of an electrical circuit, whose operational values can differ vastly from their assumed values and are often a function of the operating conditions, e.g. the impedance of a circuit element may depend on its temperature. Despite these intrinsic and extrinsic uncertainties, we want our system to behave in a certain way, in the sense of achieving certain well-defined metrics of "performance". Feedback, perhaps in the form of a PID controller of Eq.(6), allows us to fulfill this objective, thereby making the system robust against internal as well as external uncertainties.

2. An important use of feedback is to ***fundamentally*** **alter the dynamical behavior** of a system. Unstable systems (such as an inverted pendulum, or one of its more physical realizations: a powered rocket!) can be stablized, systems with sluggish response can be made agile (e.g. the *stiffness* of an aircraft can be reduced), systems with drifting operating points can be held constant to operate in a desired region within its performance envelope, etc. This feature of feedback is sometimes called *design of dynamics* (Åström & Murray). Such "dynamics design" also serves to increase modularity of the overall system. By essentially controlling the system to have a desired overall dynamic profile, we can mask the complexities and variability in its subsystems, precluding the need to tune each individual such subsystem to achieve desired behavior.

3. Feedback, in recent years, has helped up achieve unparalleled **autonomy** of dynamical systems. For a system (called *agent* henceforth) to operate autonomously, i.e. unsupervised by humans, it must have so-called situational awareness and decision making capability in a potentially unknown, unstructured environment. This invokes several streams of inquiry that are usually encountered in the community of artificial intelligence, e.g. learning, adaptation, and even abstract reasoning, all with some sense of optimality. There is an increasing role of dynamics, robustness and interconnection in these fields, leading to new branches of control such as distributed control and cooperative control involving multi-agent autonomous teams. A commonly cited example is autonomous cars, which are now reaching a reasonable level of maturity, e.g. the vehicles that participate in the DARPA Grand Challenge.

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